

CAN LIGO SEE COMPACT BINARIES?

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February 6, 2008

Abstract

The probability that interferometric detectors such as LIGO and VIRGO will successfully detect inspiraling compact binaries depends in part on our knowledge of the expected gravitational wave forms.

The best approximations to the true wave forms available today are the post-Newtonian (PN) templates. In this paper we argue that these 2PN templates are accurate enough for a successful search for compact binaries with the advanced LIGO interferometer. Results are presented for the 40-meter Caltech prototype as well as for the initial and advanced LIGO detectors.

1 INTRODUCTION

Currently (November 1997) there are five large scale interferometric gravitational-wave detectors under construction, namely the two American LIGO detectors, the French Italian VIRGO, the German-British GEO600 and finally the Japanese TAMA. The immediate goal of this effort is the direct detection of gravitational waves. In a later stage this international network of detectors will act as a gravitational wave observatory with a variety of scientific projects (see reference [1] for more details). The most promising sources for detection are inspiraling binaries of black holes and/or neutron stars.

Because since gravitational waves are so weak, any signal will likely be obscured by detector noise. The method of choice to search for a *known* signal in a noisy data stream is matched filtering [2]: The data is compared to a set of template wave forms for a “best fit” (a more precise definition is given below). Matched filtering is an optimal method in the sense that no other (linear) filtering method is more effective. The drawback is that a set of very accurate templates is needed to track the signal with the necessary accuracy. For example a typical inspiraling-binary signal might approximately 15000 wave cycles in the advanced LIGO frequency band. This means that we need a template that traces all of the 15000 cycles. By missing only one cycle the detection probability falls drastically. Unfortunately,

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in general relativity there is no way to (analytically) calculate the exact wave-form and we have to resort to some kind of approximation. The most promising approximative templates are obtained by a post-Newtonian expansion of the gravitational waves in the orbital velocity of the binary companions.

In this contribution we assess the quality of the PN templates by comparing them to signals obtained from black hole perturbation theory. This is known to provide very accurate wave forms when the binary mass ratio is small.

2 MATCHED FILTERING

Let us write the detector output as

$$s(t) = h(t) + n(t),$$

where $n(t)$ denotes the detector noise and $h(t)$ a possibly absent signal. We define the squared signal to noise ratio (SNR) by

$$\rho^2 = \frac{(s, t)}{\text{rms}(n, t)}. \quad (1)$$

Here the inner product $(\ , \)$ is defined by

$$(s, t) := 2 \int_0^\infty \frac{df}{S_n(f)} \left(\hat{s}(f) \hat{t}^*(f) + \hat{s}^*(f) \hat{t}(f) \right), \quad (2)$$

where a hat $\hat{}$ denotes a Fourier transform and an asterisk $*$ complex conjugation. S_n is the (one sided) spectral density of the noise. It is large where the noise is large. Consequently, regions of low noise get a higher weight in the integral (2), the “match” between the template t and the detector output h . If in an actual measurement ρ exceeds a certain threshold then detection is claimed.

Wieners theorem [2] now states that ρ is maximal if $t = h$ in equation (1). This choice of template is called the Wiener optimal filter. Conversely, if $t \neq h$ then we get a smaller ρ and thus might miss signals that otherwise would have been detected.

Since we want to make statements about the templates for “typical” noise, we usually talk about the *expectation value* of the SNR. Under the assumption of Gaussian noise it can be calculate to be [3]

$$\langle \rho^2 \rangle = \frac{(h, t)}{\underbrace{\sqrt{(t, t)(h, h)}}_{\mathcal{A}}} \langle \rho^2 \rangle_{\text{best}}.$$

The quantity $\langle \rho^2 \rangle_{\text{best}} = \sqrt{(h, h)}$ is the SNR one would obtain if one knew the exact waveform h . The function \mathcal{A} is called the *ambiguity function* and it depends on the template used. Maximizing \mathcal{A} over all templates gives the *fitting factor* FF . This is the reduction of the SNR caused by inaccurate templates: $\langle \rho^2 \rangle_{\text{max}} = FF \langle \rho^2 \rangle_{\text{best}}$. The fitting factor therefore measures the quality of the templates.

The best templates available today are calculated using a post-Newtonian expansion [4] of Einstein’s equations. For inspiraling binaries, the wave forms are expressed as a power series in v/c where v is the orbital velocity. (Recently Padé approximates have been used to improve the convergence of the post-Newtonian expansion [5]).

To simplify matters, one often works with the so-called restricted post-Newtonian approximation given by

$$t(f) \propto f^{-\frac{7}{3}} e^{i\Psi(\mathcal{M}, \eta, t_c, \phi_c)}, \quad (3)$$

where the amplitude is kept at the lowest post-Newtonian order but the phase $\Psi(\mathcal{M}, \eta, t_c, \phi_c)$ includes higher PN corrections. Here we keep Ψ accurate to 2 PN order. This makes sense, because we expect a slight mismatch in the phases to lead to a rapidly oscillating integrand in (2) and thus to a low SNR. Here the chirp mass $\mathcal{M} = (m_1 m_2)^{\frac{3}{5}} / (m_1 + m_2)^{\frac{1}{5}}$, the mass ratio $\eta = (m_1 m_2) / (m_1 + m_2)^2$, the arrival time t_c and the initial phase ϕ_c parameterize the templates.

The test signal used to calculate the fitting factor is obtained from black hole perturbation theory [6], integrating the Teukolsky equation [7]. The signal is given as an infinite sum over modes and takes the form [8]

$$h(f) \propto \sum_{l=2}^{\infty} \sum_{m=1}^l A^{lm}(v) e^{\psi(v)} S_l(\vartheta, \varphi) \quad (4)$$

where $v = (2\pi M f / m)^{1/3}$ ($M = m_1 + m_2$). The functions $A_{lm}(v)$ and $\psi(v)$ must be calculated numerically and S_l is a known function. The $l = m = 2$ mode dominates the signal and we have verified numerically that modes with $l > 3$ can be neglected in the sum (4).

To calculate the fitting factor one sticks expressions (3) and (4) into the definition of the ambiguity function and maximizes over the parameters \mathcal{M} , η , t_c and ϕ_c for a “fixed” signal (4).

3 RESULTS AND INTERPRETATION

System	advanced LIGO		initial LIGO	40-meter
	FF -Newton	FF - 2-PN	FF - 2-PN	FF - 2-PN
$1.4 - 1.4 M_{\odot}$	79.2%	93.0%	95.8%	94.0%
$0.5 - 5.0 M_{\odot}$	51.6%	95.2%	89.7%	89.0%
$1.4 - 10 M_{\odot}$	55.8%	91.4%	88.4%	< 85.0%
$10 - 10 M_{\odot}$	70.1%	91.7%	86.3%	
$4 - 30 M_{\odot}$	61.3%	86.6%	67.9%	

Table 1: The fitting factors for different noise curves and binary systems are listed. A fitting factor larger than 90% is considered acceptable. Note that only the $1.4 - 1.4 M_{\odot}$ system, for which $FF > 90\%$, is within the targeted range of the 40-meter detector.

We have calculated the fitting factors for a variety of different binaries and detector noise curves (for the initial and advanced LIGO detectors as well as the 40m Caltech prototype).

In the former case, in addition to the 2 PN templates we have also tested the Newtonian (\equiv lowest PN) templates. The results are summarized in Table 1.

We find that for the mass range of interest to a particular detector, the fitting factors for the 2PN templates are all above 90%. This makes them suitable for detection of inspiraling binary signal [9]. For small mass ratios the fitting factors drop considerably reflecting the fact that the systems become more relativistic for fixed chirp mass \mathcal{M}_c . One would expect PN templates to perform less well in this limit.

An additional point should be made. Formally, black hole perturbation theory assumes $\eta \equiv 0$. Thus it is not clear if our analysis holds for nearly-equal-mass systems. However, by setting $\eta = 0$ in the templates (3) one effectively obtains a PN expansion of the perturbation-theory signal. When we do so we do not find any significant changes in the fitting factor. This leads us to believe that our results should hold, at least qualitatively, even for nonzero mass ratios.

In conclusion, we may say that the 2 PN templates are suitable for the detection of binary inspiral signals.

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